# Global Optimality
# for Optimal Power Flow Problem

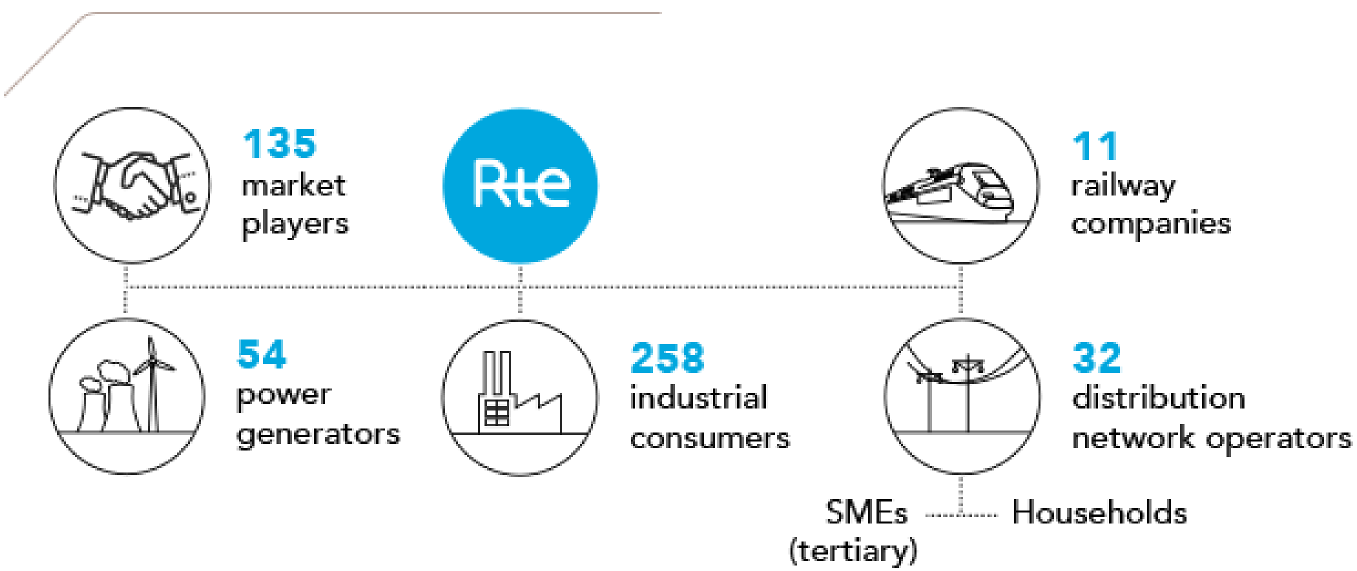
**Spring School on Mixed Integer Nonlinear Programming and Applications**

April 4th 2016, Jean Maeght - RTE

# RTE Réseau de Transport d'Electricité

- 8500 staff - 105000 km of lines 63 to 400kV
- 48 cross-border connections – largest TSO in Europe
- 100% subsidiary of EDF
- In charge of the grid but also heart of the power *system*

**Who are our 490 customers?**



| Longueur de circuits en exploitation (km) | Total (courant alternatif) | | | Total (courant continu) |
|---|---|---|---|---|
| | Aérien | Souterrain | Total | |
| Au 31/12/2014 | 100 610 | 4 721 | 105 331 | 117 |
| Neuf | 283 | 342 | 625 | 66 |
| *nouveau* | *20* | *237* | *257* | *66* |
| *renouvelé* | *263* | *17* | *280* | *0* |
| *aérien mis en souterrain* | *0* | *88* | *88* | *0* |
| Ferraillé | -394 | -2 | -396 | 0 |
| Autres modifications (corrections de longueur) | -87 | -25 | -112 | 0 |
| Au 31/12/2015 | 100 412 | 5 036 | 105 448 | 183 |
| Ecarts 2014 - 2015 | -198 | 315 | 117 | 66 |

# Would we have some work without grid ?

- Imagine that no grid is needed for power transmission
  World is a ideal copper plate
- Electricity storage is still difficult
  Balance 100% of the time. Or frequency drops
- What happens when wind power drops? When solar power increase suddenly?
- Financial vs. technical responsibility

# R&D in RTE

- 120+ staff members. R&D Showroom
  R&D activity is now regulated
- Main network analysis tool (simulation) in developed and maintained internally (400+ users)
- Many others: economics, adequacy, long term and short term load forecasting, grid developement
- Strong European Cooperation (EU projects)

# Optimization problem

- Minimize something
- With a representation of the state of the grid
  - Static – stationary state
  - Alternative Current (AC) modelling – product of complex numbers
  - Technical constraints:
    Minimal and maximal Voltage values
    Thermal limits
    Generation capabilities (minP maxP)
    Equipments: lines, transformers, shunts, etc…

# Optimal Power Flow

- Carpentier (1962). Contribution à l'étude du dispatching économique. Bulletin de la Société Française des Électriciens

- Minimization of linear generation costs
  With representation of the state of the grid
- Simplest case: all costs equal 1

- Many variants: objective function, variables (e.g. slack variables for unfeasibility detection)

# Variables and data (Cédric Josz's thesis)

| objective | description |
|---|---|
| $\min \sum_{k\in\mathcal{G}} a_k(p^{\text{gen}})^2 + b_k p^{\text{gen}} + c_k$ | generation cost |
| **variables** | **description** |
| $(i_k)_{k\in\mathcal{N}}$ | injected current |
| $(i_{lm})_{(l,m)\in\mathcal{L}}$ | current flow |
| $(p_k^{\text{gen}})_{k\in\mathcal{N}}$ | active generation |
| $(q_k^{\text{gen}})_{k\in\mathcal{N}}$ | reactive generation |
| $(v_k)_{k\in\mathcal{N}}$ | voltage |
| **data** | **description** |
| $(y_{lm})_{(l,m)\in\mathcal{L}}$ | mutual admittance |
| $(y_{lm}^{\text{gr}})_{(l,m)\in\mathcal{L}}$ | admittance-to-ground |
| $(\rho_{lm})_{(l,m)\in\mathcal{L}}$ | ratio of (phase-shifting) transformer |
| $(p_k^{\text{dem}})_{k\in\mathcal{N}}$ | active power demand |
| $(q_k^{\text{dem}})_{k\in\mathcal{N}}$ | reactive power demand |
| $v_k^{\min}, v_k^{\max}, p_k^{\min}, p_k^{\max}, q_k^{\min}, q_k^{\max}$ | bounds at buses |
| $i_{lm}^{\max}, v_{lm}^{\max}, s_{lm}^{\max}, p_{lm}^{\max}$ | bounds on line flow |

# Constraints (Cédric Josz's thesis)

| $k \in \ldots$ <br> $(l,m) \in \ldots$ | **constraints** | **description** |
|---|---|---|
| $\mathcal{N}$ | $i_l = \sum_{m \in \mathcal{N}(l)} i_{lm}$ | Kirchoff's first law |
| $\mathcal{L}$ | $\rho_{lm}^H i_{lm} = y_{lm}^{\text{gr}} \frac{v_l}{\rho_{lm}} + y_{lm}(\frac{v_l}{\rho_{lm}} - \frac{v_m}{\rho_{ml}})$ | Kirchoff's first law and Ohm's law |
| $\mathcal{N} \setminus \mathcal{G}$ | $v_k i_k^H = -p_k^{\text{dem}} - \text{j} q_k^{\text{dem}}$ | power demand |
| $\mathcal{G}$ | $v_k i_k^H = p_k^{\text{gen}} - p_k^{\text{dem}} + \text{j}(q_k^{\text{gen}} - q_k^{\text{dem}})$ | power demand and generation |
| $\mathcal{G}$ | $p_k^{\min} \leqslant p_k^{\text{gen}} \leqslant p_k^{\max}$ | bounds on active generation |
| $\mathcal{G}$ | $q_k^{\min} \leqslant q_k^{\text{gen}} \leqslant q_k^{\max}$ | bounds on reactive generation |
| $\mathcal{N}$ | $v_k^{\min} \leqslant \lvert v_k \rvert \leqslant v_k^{\max}$ | bounds on voltage amplitude |
| $\mathcal{L}$ | $\lvert v_l - v_m \rvert \leqslant v_{lm}^{\max}$ | bound on voltage difference |
| $\mathcal{L}$ | $\lvert i_{lm} \rvert \leqslant i_{lm}^{\max}$ | bound on current flow |
| $\mathcal{L}$ | $\lvert v_l i_{lm}^H \rvert \leqslant s_{lm}^{\max}$ | bound on apparent power flow |
| $\mathcal{L}$ | $\lvert \text{Re}(v_l i_{lm}^H) \rvert \leqslant p_{lm}^{\max}$ | bound on active power flow |

# Application of the Moment-SOS Approach to Global Optimization of the OPF Problem

C. Josz, J. M., P. Panciatici, J.-C. Gilbert

- WB5 problem using GloptiPoly version 3.6.1

| $q_5^{\min}$ (MVA) | relax. order | optimal value ($/h) | rank relax. value ($/h) |
|---|---|---|---|
| -30.80 | 2 | 945.83 | 945.83 |
| -20.51 | 2 | 1146.48 | (954.82) |
| -10.22 | 2 | 1209.11 | (963.83) |
| 00.07 | 2 | 1267.79 | (972.85) |
| 10.36 | 2 | 1323.86 | (981.89) |
| 20.65 | 2 | 1377.97 | (990.95) |
| 30.94 | 2 | 1430.54 | (1005.13) |
| 41.23 | 2 | 1481.81 | (1033.07) |
| 51.52 | 2 | 1531.97 | (1070.39) |
| 61.81 | - | - | (1114.90) |

## Moment/Sum-of-Squares Hierarchy for Complex Polynomial Optimization
## Cédric Josz, Daniel K. Molzahn 2015
## (losses minimization)

| Case Name | Num. Iter. | Global Obj. Val. (MW) | Max $S^{mis}$ (MVA) | Solver Time (sec) |
|---|---|---|---|---|
| PL-2383wp | 3 | 24,991 | 0.10 | 53.9 |
| PL-2736sp | 1 | 18,335 | 0.11 | 17.8 |
| PL-2737sop | 1 | 11,397 | 0.07 | 25.7 |
| PL-2746wop | 2 | 19,212 | 0.12 | 124.3 |
| PL-2746wp | 1 | 25,269 | 0.05 | 18.5 |
| PL-3012wp | 7 | 27,644 | 0.91 | 141.0 |
| PL-3120sp | 9 | 21,512 | 0.27 | 193.9 |
| PEGASE-1354 | 11 | 74,042 | 1.00 | 1,132.6 |
| PEGASE-2869 | 9 | 133,939 | 0.97 | 700.8 |

# AC Power Flow Data in MATPOWER and QCQP Format: iTesla, RTE Snapshots, and PEGASE
## Josz, Fliscounakis, M., Panciatici 2016

| Losses minimization without flow constraints | | | | |
|---|---|---|---|---|
| Algorithm: | DCOPF | SDPOPF | SDPOPF | OPF |
| Solver: | Mips | Sedumi | Mosek | Knitro |
| case89pegase | 5 733.4 | 5 817.6 | 5 817.6 | 5 817.6 |
| case1354pegase | 73 059.7 | 74 052.8 | 74 049.5 | 74 060.4 |
| case1888rte | 59 110.5 | 59 572.0 | 59 557.7 | 59 769.9 |
| case1951rte | 80 656.5 | 81 718.7 | 81 706.4 | 81 724.2 |
| case2848rte | 52 562.3 | 53 006.6 | 52 986.4 | 53 020.9 |
| case2868rte | 78 826.3 | 79 782.9 | 79 769.1 | 79 783.4 |
| case2869pegase | 132 447.2 | 133 970.9 | 133 964.6 | 133 980.7 |
| case6468rte | 85 296.9 | 86 754.5 | 86 726.2 | 86 791.8 |
| case6470rte | 96 592.4 | 98 305.0 | 98 277.0 | 98 308.0 |
| case6495rte | 103 916.1 | 105 969.7 | 105 919.4 | 105 943.6 |
| case6515rte | 107 264.0 | 109 560.7 | 109 533.5 | 109 561.2 |
| case9241pegase | 312 411.0 | 310 723.5 | 310 697.1 | 315 888.5 |
| case13659pegase | 381 773.4 | 381 047.8 | 381 027.7 | 386 107.5 |

# Optimality achievement?

| Global Optimality proofs | | |
|---|---|---|
| | DCOPF | SDPOPF |
| case89pegase | 1.47% | 0.00% |
| case1354pegase | 1.37% | 0.01% |
| case1888rte | 1.12% | 0.36% |
| case1951rte | 1.32% | 0.02% |
| case2848rte | 0.87% | 0.07% |
| case2868rte | 1.21% | 0.02% |
| case2869pegase | 1.16% | 0.01% |
| case6468rte | 1.75% | 0.08% |
| case6470rte | 1.78% | 0.03% |
| case6495rte | 1.95% | 0.02% |
| case6515rte | 2.14% | 0.03% |
| case9241pegase | not valid | 1.67% |
| case13659pegase | not valid | 1.33% |

# Conclusion: Optimality Quest

- Although losses minimization and Optimal dispatch are not direct targets for RTE's business
- Variants of OPFs already used on daily basis, progress for standard OPFs to be translated to variants.
- Introduction of integer variables
- Inclusion in larger simulation processes, e.g.
  - Alexander Mitsos. Global solution of nonlinear mixed-integer bilevel programs. *Journal of Global Optimization*, 2009
    Thesis in progress, Aachen
- Need for more efficient SDP solvers

# Starting References

- Computational Analysis of Sparsity-Exploiting Moment Relaxations of the OPF Problem
  Molzahn, Hiskens, Josz, Panciatici, 2016 (arXiv)
  Introduction gives a clear history of SDP methods for OPF, with references

- AC Power Flow Data in MATPOWER and QCQP Format: iTesla, RTE Snapshots, and PEGASE
  Josz, Fliscounakis, M., Panciatici, 2016 (arXiv)
  Large datasets with computational results